

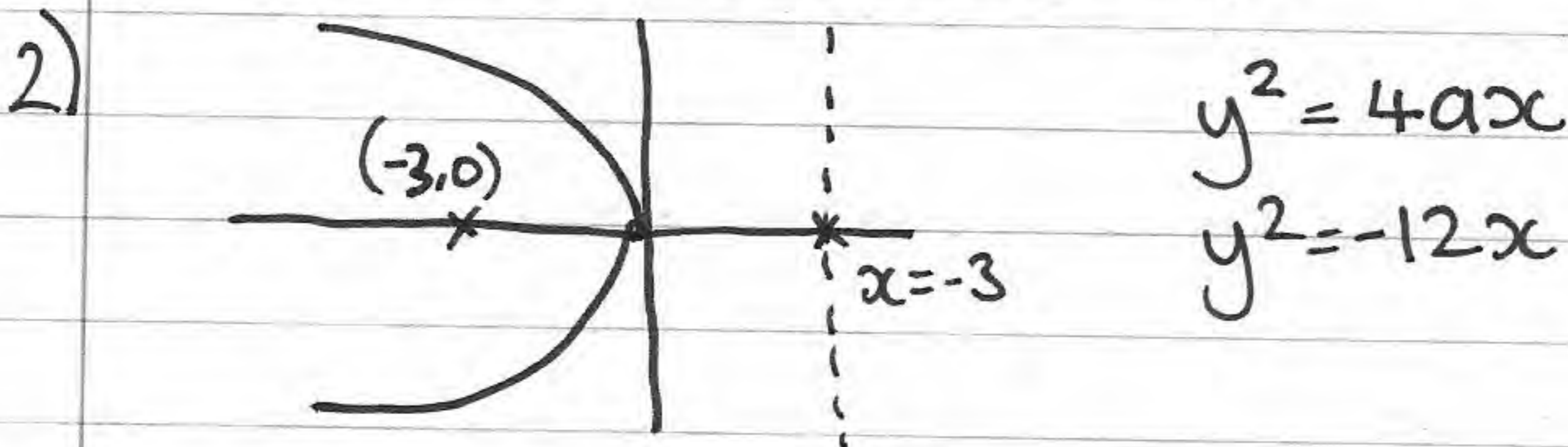
FP1 Mock Paper

1) $R = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $S = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

a) $R^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

b) $RS = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

rotation 180° about origin



3) $Z = 1 + i\sqrt{3}$ $Z^2 = (1 + i\sqrt{3})(1 + i\sqrt{3})$
 $= 1 + 3i^2 + 2\sqrt{3}i$
 $= -2 + i2\sqrt{3}$

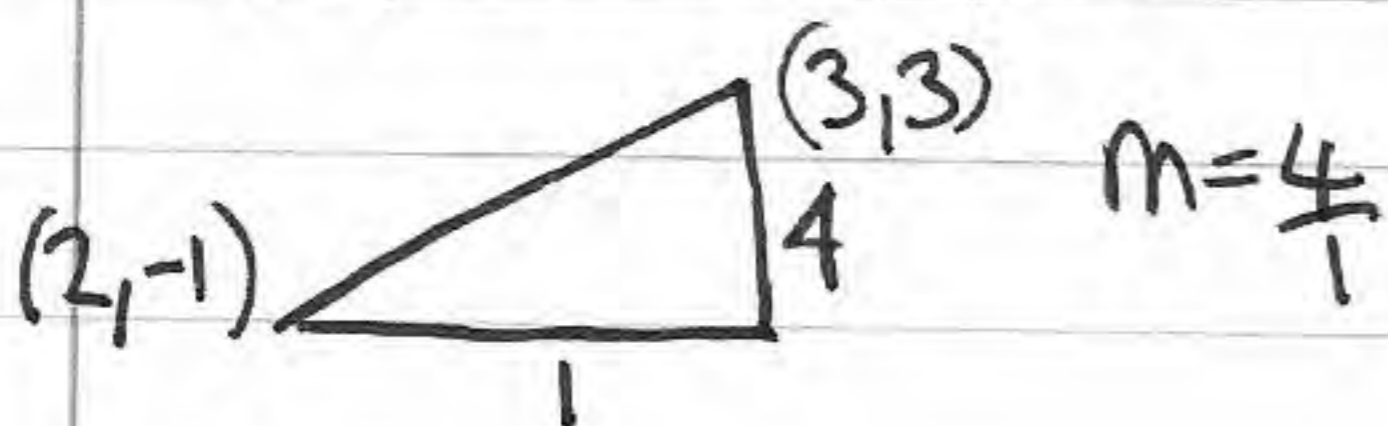
$$\begin{aligned} Z + Z^2 &= 1 + i\sqrt{3} \\ &\quad -2 + i2\sqrt{3} \\ \hline &= -1 + i3\sqrt{3} \end{aligned}$$

b) $\frac{Z}{3-2} = \frac{1+i\sqrt{3}}{3-1-i\sqrt{3}} = \frac{1+i\sqrt{3} \times (2+i\sqrt{3})}{2-i\sqrt{3} \times (2+i\sqrt{3})}$
 $= \frac{2+3i^2+i3\sqrt{3}}{4-3i^2} = \frac{-1+i3\sqrt{3}}{7} = -\frac{1}{7} + i\frac{3\sqrt{3}}{7}$

4) $f(x) = x^3 - 4x^2 + 5x - 3$ $f(2) = -1$ $f(3) = 3$

$(2, -1) \rightarrow (3, 3)$

$y+1 = 4(x-2)$



$y=0 \Rightarrow \frac{1}{4} = x-2 \Rightarrow x = 2\frac{1}{4}$

b) $f'(x) = 3x^2 - 8x + 5$

$$x_0 = 2.5 \quad x_1 = 2.5 - \frac{f(2.5)}{f'(2.5)} = \frac{37}{15} = \underline{2.47} \text{ (2dp)}$$

5) $X = \begin{pmatrix} a & 2b \\ -a & 3b \end{pmatrix} \quad \det X = 3ab + 2ab = 5ab$

$$X^{-1} = \frac{1}{5ab} \begin{pmatrix} 3b & -2b \\ a & a \end{pmatrix}$$

b) $ZX = Y \Rightarrow ZX X^{-1} = Y X^{-1} \Rightarrow Z = Y X^{-1}$

$$Z = \begin{pmatrix} 3a & b \\ a & 2b \end{pmatrix} \begin{pmatrix} 3b & -2b \\ a & a \end{pmatrix} \frac{1}{5ab} = \frac{1}{5ab} \begin{pmatrix} 9ab + ab & -6ab + ab \\ 3ab + 2ab & -2ab + 2ab \end{pmatrix}$$

$$Z = \frac{1}{5ab} \begin{pmatrix} 10ab & -5ab \\ 5ab & 0 \end{pmatrix} \Rightarrow Z = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$$

6) $\sum_1^n r(2r^2 - 6) = 2 \sum_1^n r^3 - 6 \sum_1^n r = 2 \left(\frac{1}{4} n^2 (n+1)^2 \right) - 6 \left(\frac{1}{2} n(n+1) \right)$

$$= \frac{1}{2} n(n+1) [n(n+1) - 6] = \frac{1}{2} n(n+1)(n^2 + n - 6)$$

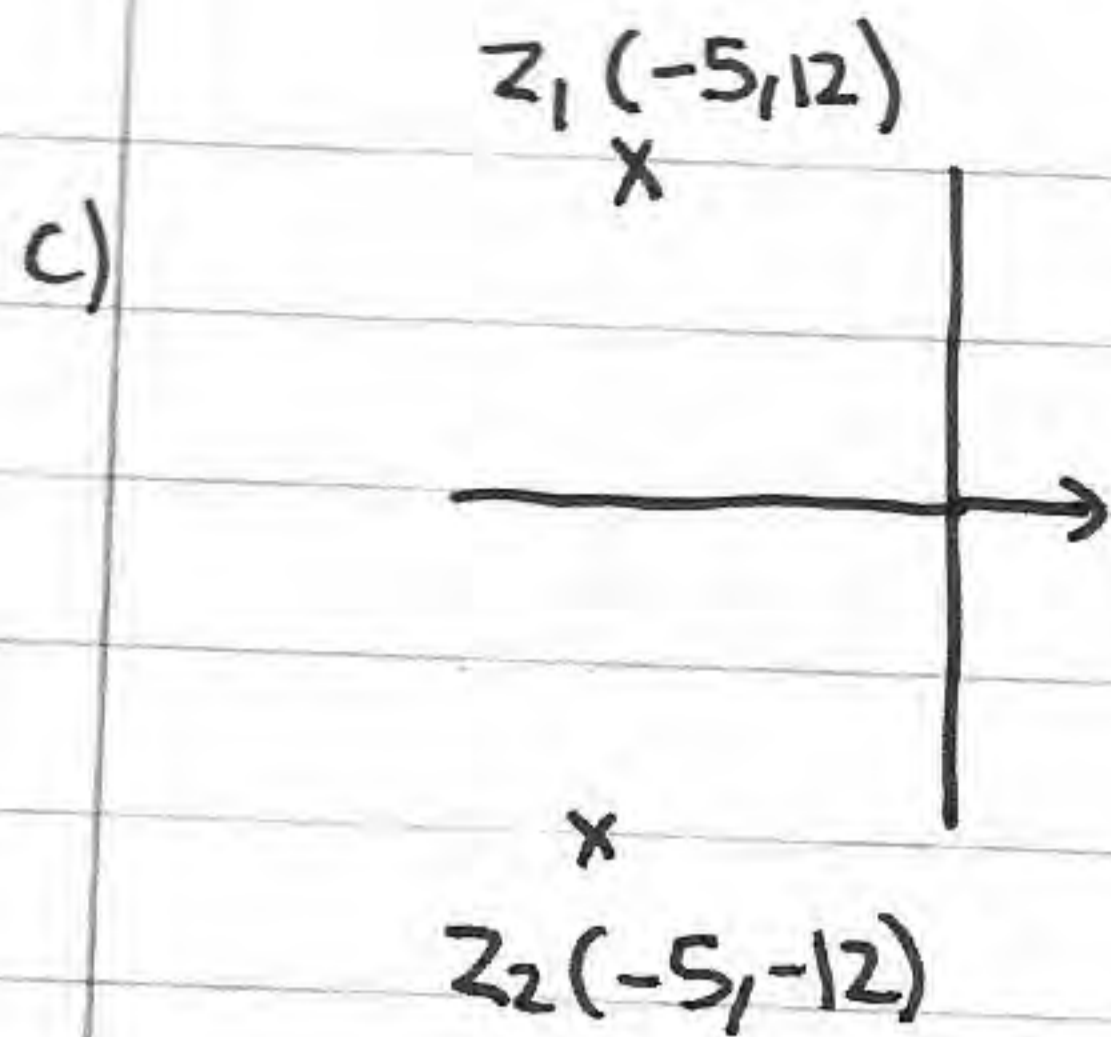
$$= \frac{1}{2} n(n+1)(n+3)(n-2) \quad \#$$

b) $\sum_{10}^{50} r(2r^2 - 6) = \frac{1}{2} (50)(51)(53)(48) - \frac{1}{2} (9)(10)(12)(7) = \underline{3239820}$

7) $Z^2 + 10Z + 169 = 0 \Rightarrow (Z+5)^2 - 25 = -169 \Rightarrow (Z+5)^2 = -144$

$$\Rightarrow Z = -5 \pm \sqrt{-144} \Rightarrow Z_1 = \underline{-5 + 12i}, Z_2 = \underline{-5 - 12i}$$

b) $|Z_1| = |Z_2| = \sqrt{5^2 + 12^2} = 13$



$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{12}{-5} \Rightarrow \theta = \tan^{-1}\left(\frac{12}{-5}\right)$$

$$\theta = -1.176$$

$$+\pi$$

$$\arg(z_1) = 1.97$$

$$\Rightarrow \arg(z_2) = -1.97$$

d) $|z_1 - z_2| = |-5 + 12i - (-5 - 12i)| = |24i| = \underline{24}$

8) $xy = c^2$ $\left(3t, \frac{3}{t}\right) \Rightarrow 3t\left(\frac{3}{t}\right) = c^2 \Rightarrow \underline{9 = c^2}$ $c = \pm 3$

b) $xy = 9$ $y = 9x^{-1} \Rightarrow \frac{dy}{dx} = -9x^{-2} = \frac{-9}{x^2} = \frac{-9}{9t^2}$

$m_t = \frac{-1}{t^2} \Rightarrow m_n = t^2$ $y - \frac{3}{t} = t^2(x - 3t)$

$\Rightarrow y - \frac{3}{t} = t^2x - 3t^3 \therefore y = t^2x + \left(\frac{3}{t} - 3t^3\right)$

$$c) H(6, 1.5) = (3t, \frac{3}{t}) \Rightarrow t=2 \Rightarrow y = 4x + (\frac{3}{2} - 3(2)^3)$$
$$y = 4x - \frac{45}{2}$$

$$xy = 9 \Rightarrow x(4x - \frac{45}{2}) = 9 \Rightarrow 4x^2 - \frac{45}{2}x - 9 = 0$$

$$\Rightarrow 8x^2 - 45x - 18 = 0 \quad (x-6)(8x+3) = 0$$

$$x=6 \vee x = -\frac{3}{8} \quad y = -\frac{12}{8} - \frac{45}{2} = -24 \quad Q(-\frac{3}{8}, -24)$$

a) $U_1 = 3$ $U_{n+1} = 3U_n + 4$ prove $U_n = 3^n + 2(3^{n-1} - 1)$

$n=1$ $U_1 = 3$ $U_1 = 3^1 + 2(3^{1-1} - 1) = 3^1 = 3$ ✓

$n=2$ $U_2 = 3(3) + 4 = 13$ $U_2 = 3^2 + 2(3^{2-1} - 1) = 9 + 4 = 13$ ✓

$n=k+1$ $U_{k+1} = 3U_k + 4 = 3(3^k + 2(3^{k-1} - 1)) + 4$

$\Rightarrow U_{k+1} = 3^{k+1} + 2[3(3^{k-1} - 1)] + 4$

$\Rightarrow U_{k+1} = 3^{k+1} + 2(3^k - 3) + 2 \times 2$

$\Rightarrow U_{k+1} = 3^{k+1} + 2(3^k - 3 + 2)$

$\therefore U_{k+1} = 3^{k+1} + 2(3^k - 1)$

true for $n=1, n=2$; true for $n=k+1$, if true for $n=k$
 \therefore by induction, true for all $n \in \mathbb{Z}^+$.

b) $A = \begin{pmatrix} 4 & 0 \\ 9 & 1 \end{pmatrix}$ prove $A^n = \begin{pmatrix} 4^n & 0 \\ 3(4^n - 1) & 1 \end{pmatrix}$

$n=1$ $A^1 = A = \begin{pmatrix} 4^1 & 0 \\ 3(4^1 - 1) & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 9 & 1 \end{pmatrix}$ ✓

$n=k+1 \Rightarrow A^{k+1} = \begin{pmatrix} 4^{k+1} & 0 \\ 3(4^{k+1} - 1) & 1 \end{pmatrix}$

$A^{k+1} = A A^k = \begin{pmatrix} 4 & 0 \\ 9 & 1 \end{pmatrix} \begin{pmatrix} 4^k & 0 \\ 3(4^k - 1) & 1 \end{pmatrix} = \begin{pmatrix} 4 \times 4^k & 0 \\ 9 \times 4^k + 3(4^k - 1) & 1 \end{pmatrix}$

$9(4^k) + 3(4^k - 1) = 12(4^k) - 3 = 3 \times 4(4^k) - 3 = 3(4^{k+1} - 1)$

true for $n=1$, true for $n=k+1$ if true for $n=k$
 \therefore by induction true for all $n \in \mathbb{Z}^+$

$$(ii) \quad A^{-1} \begin{pmatrix} 4^{-1} & 0 \\ 3(4^{-1}-1) & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & 0 \\ 3(\frac{1}{4}-1) & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & 0 \\ -\frac{9}{4} & 1 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 & 0 \\ -9 & 4 \end{pmatrix}$$

$$\det A = 4 - 0 = 4 \quad A^{-1} = \frac{1}{\det A} \begin{pmatrix} 1 & 0 \\ -9 & 4 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 & 0 \\ -9 & 4 \end{pmatrix} \checkmark$$

\therefore It is also valid for $n = -1$